Number Ratio and Proportion - Foundation

Estimate
Round each value to one significant figure

Standard form
$a \times 10^n$, where $1 \leq a < 10$

Reciprocal
Reciprocal of 7 is $\frac{1}{7}$, reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ etc

Sequences
Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21
Geometric Sequence: each term is multiplied but he same constant to get the next number.
E.g. 3, 12, 48, 191, …. (x by 4 each time)

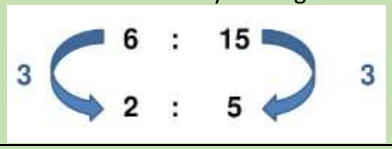
Squares and Cubes
Square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225 etc

Cube numbers: 1, 8, 27, 64, 125, 216, 343, 512, etc

Sharing in a given Ratio
A Add the ratio parts
D Divide the amount by the total parts
A and
M Multiply the ratio by the value of one part

e.g. share £420 in the ratio 2:5
2 + 5 = 7
420 ÷ 7 = £60
2: 5
(x60) (x60)
£120 : £300

Simplifying Ratio
Divide both sides by the highest common factor



Simplifying Ratio 1:n
Divide both sides by the highest factor of the left hand side
2m: 180cm
200cm: 180cm
2:1.8
1: 0.9

Fractions

Add and Subtract – ensure the fractions have the same denominator before adding numerators

$$\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$$

Multiply – multiply numerators and denominators

$$\frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$$

Divide – take reciprocal of the second fraction and then multiply the new numerators and denominators

$$\frac{4}{5} \div \frac{1}{3} = \frac{4}{5} \times \frac{3}{1} = \frac{12}{5} = 2\frac{2}{5}$$

Percentages

Finding percentages of an amount
1% ÷100
5% ÷20
20% ÷5
25% ÷4
50% ÷2

Multipliers:
To find the multiplier for a percentage, divide by 100

Use multipliers on a calculator paper
e.g. 35% of 370 = 0.35 x 370

Increasing and decreasing a given amount
Calculator:

$$Orginal\ Amount\ x\ mutiplier = new\ amount$$

Non-calculator: find the increase or decrease and add to the original amount

Finding percentage increase or decrease (profit/loss)

$$\frac{value\ of\ increase/decrease}{Original} \times 100$$

Writing an amount as a percentage of the original

$$\frac{Amount}{Original} \times 100$$

Reverse Percentage – finding the original amount

$$Orginal\ Amount = \frac{New\ Amount}{multiplier}$$

Growth & Decay / Compound interest

$$original\ amount \times multiplier^{time}$$

Where the multiplier is the percentage, increase or decrease from 100%, converted to a decimal.
e.g.
30% decrease is 70% = 0.7
30% increase is 130% = 1.3

Dividing by decimals:

1. Write the calculation as a fraction
2. Form an equivalent fraction to makes integers (multiply by powers of 10)
3. Use short division (bus stop) to calculate

e.g. $460 \div 0.4 = \frac{460}{0.4} = \frac{4600}{4} = 1150$

Conversions

10 millimetres = 1 centimetre	15 minutes = 0.25 hours
100 centimetres = 1 metre	30 minutes = 0.5 hours
1000 metres = 1 kilometre	45 minutes = 0.75 hours
$1000cm^3$ = 1 litre	1000g = 1 kilogram
1000ml = 1 litre	1000kg = 1 tonne

Compound Units (rearrange as necessary)

$$Speed = \frac{Distance}{Time}$$

$$Area = \frac{Force}{Pressure}$$

$$Density = \frac{Mass}{Volume}$$

Error Intervals
least possible value $\leq x <$ greatest possible value

e.g. A fence is 30 m long to the nearest 10 m.
$25\text{ m} \leq l < 35\text{ m}$

Truncation
Truncation is a method of approximating a decimal number by dropping all decimal places past a certain point without rounding.

e.g. Truncate 3.14159265 to 4 decimal places.
= 3.1415

Negative numbers
<u>Adding and subtracting: (vertical number lines help)</u>
-3 – 5 = - 8
-3 + 5 = 2
-3 - - 5 = -3 + 5 = 2
-3 - + 5 = -3 – 5 = -8
-3 + - 5 = -3 – 5 = -8

<u>Multiplying and dividing:</u>
Different signs – answer will be negative
+ x - = -, - x + = -
Same signs – answer will be positive
- x - = +

Ordering fractions
Calc: use division to write each fraction as a decimal
Non-calc: write fractions with common denominators

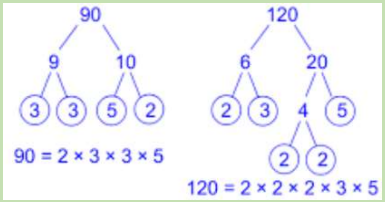
Order of operations
Bracket
Indices
Division and **M**ultiplication
Addition and **S**ubtraction

Rounding to significant figures
Start from the first **non-zero** number and round as normal, but ensure the place value is correct
e.g. 345,635 to 2SF = 350,000
0.0060821 to 3SF = 0.0608

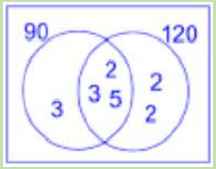
Index Laws

$$a^n \times a^m = a^{n+m}$$
$$a^n \div a^m = a^{n-m}$$
$$(a^n)^m = a^{nm}$$
$$a^0 = 1$$
$$a^{-n} = \frac{1}{a^n}$$
$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Prime Factorisation



HCF and LCM of 90 and 120 <u>(Factor Tree & Venn Diagram)</u>
HCF is the product of common factors
LCM is the product of common factors and remaining factors.



HCF: 2x3x5
LCM: 2^3x3^2x5

Notation

ab = a x b

a^2 = a x a

$(2a)^3$ = 2a x 2a x 2a

$(a + b)^2$ = (a + b)(a + b)

Definitions

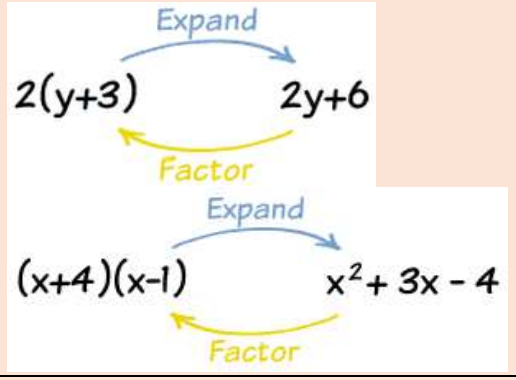
Expression – no equal signs e.g. 2x + 3, 2y, $(3x - 2)^2$

Equations – equal signs, can be solved, e.g. y + 4 = 10

Identities – identical/equivalent to e.g. 2(y + 4) ≡ 2y + 8

Formulae – equal signs, more than one unknown e.g. A= ½bh

Factorising and expanding



Straight line graphs

$y = mx + c$

$m = gradient$

$c = y - intercept$

positive gradient

negative gradient

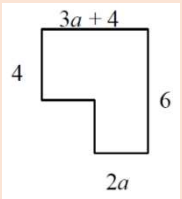
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{change\ in\ y}{change\ in\ x}$$

Parallel lines – have equal gradients

Simplifying expressions by collecting like terms

Always circle the sign IN FRONT of the term to avoid errors.

$$3x - 7b - x + 9b \equiv 2x + 2b$$



Typical Exam Q: Create an expression for the perimeter of the shape by adding and collecting like terms.

If the perimeter is given as 20cm, for example, you can create an equation:

$$4 + 3a + 4 + 6 + 2a = 20$$

$$5a + 14 = 20$$

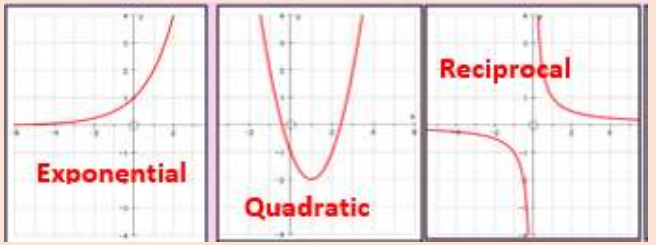
Simplifying expressions multiplication and division

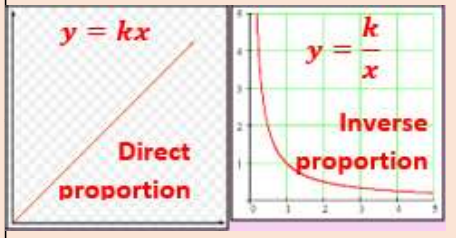
$2ma^2 \times 7ma = 14m^2a^3$

$\frac{18b^6}{3ab^2} = \frac{6b^4}{a}$

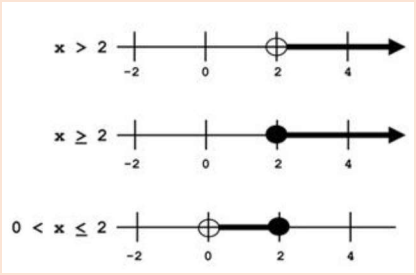
Distance / Time Graphs

Graphs that need to be recognised





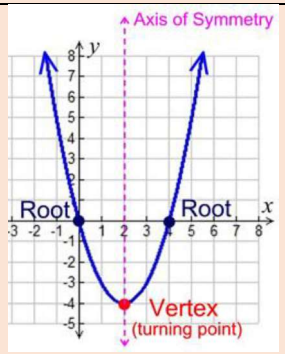
Inequalities



Open circle: </ >

Closed circle: ≤/ ≥

Turning point and roots of a quadratic equation



Finding the nth term of a linear sequence

5, 7, 9, 11, 13, ….

1. Find the common difference: 2
2. This is the coefficient of n: 2n
3. Find the difference between the coefficient of n and the first term 5 – 2 = 3
4. Add this to the amount of n

2n + 3

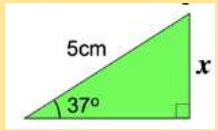
Geometry and Measures - Foundation

Trigonometry

$$S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$$

Example – finding a side:

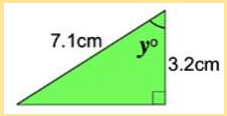
$\sin 37 = \frac{x}{5}$



x = 5 x sin 37^0

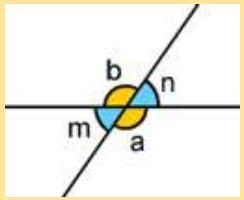
Example – finding a side:

$\tan y = \frac{3.2}{7.1}$

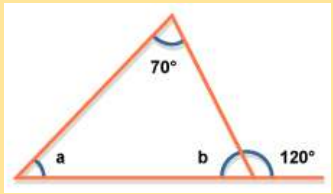


$y = tan^{-1}\left(\frac{3.2}{7.1}\right)$.

Angle Facts



Vertically opposite angles are equal: a=b and m=n

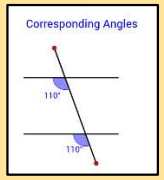


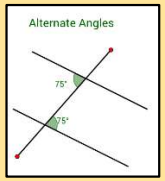
Angles in a triangle sum to 180^0.

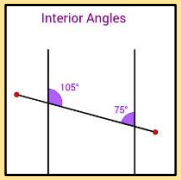
Angles on a straight line sum to 180^0.

E.G: b=60^0 so a = 50^0

Angles in parallel lines







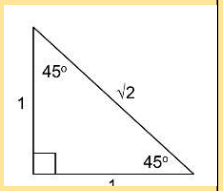
Corresponding angles are equal

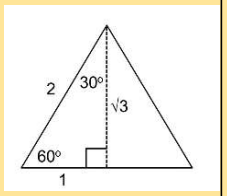
Alternate angles are equal

Co-interior angles are equal

Exact Trig values

Angle (θ)	sin(θ)	cos(θ)	tan(θ)
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined





Simple vector notation

$$\binom{a}{b}$$

a: movement along the x-axis (left or right)
b: movement along the y-axis (up or down)

$-a$: movement left $-b$: movement down

Operations with vectors

$\binom{2}{6} + \binom{7}{-3} = \binom{9}{3}$

If $b = \binom{4}{-2}$, then $3b = \binom{12}{-6}$

Volume & surface area

Volume = area of cross section x length

Surface area = area of all the faces of a 3D shape

Learn the cylinder

$$V = \pi r^2 h$$
$$SA = 2\pi r^2 + \pi dl$$

Types of triangles

Right angled
Isosceles
Equilateral
Scalene

Types of quadrilaterals

Square
Rectangle
Parallelogram
Rhombus
Trapezium
Kite

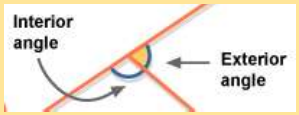
Area of key shapes

Triangle : A = $\frac{b \times h}{2}$ (h = perpendicular height)

Parallelogram: A = $b\ x\ h$ (h = perpendicular height)

Trapezium: A = $\left(\frac{a+b}{2}\right)$ x h (add together the parallel sides, divide the total by 2, and then multiply by the perpendicular height between the parallel sides)

Angles in regular polygons



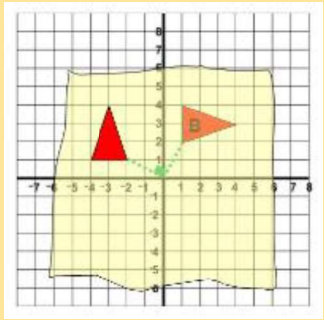
$n = number\ of\ sides$

Interior angle + exterior angle = 180^0

Exterior angle = $\frac{360}{n}$

$$n = \frac{360}{Exterior\ angle}$$

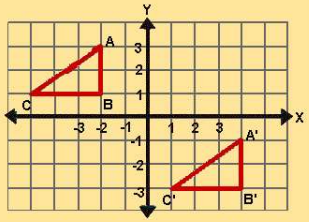
Transformations – rotation



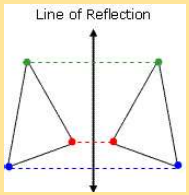
Always use tracing paper.
Describe:

1. It's a rotation
2. Size of rotation in degrees
3. Orientations: clockwise or anticlockwise
4. Centre of rotation given as a coordinate (x,y)

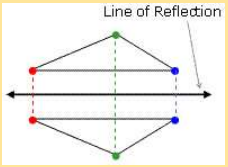
Transformations – translations and reflections



Translate triangle ABC to A'B'C' with the vector $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$

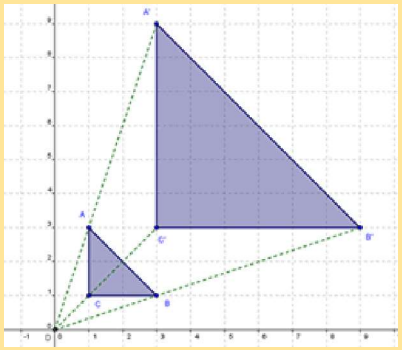


Reflection in the line x=a



Reflection in the line y=a

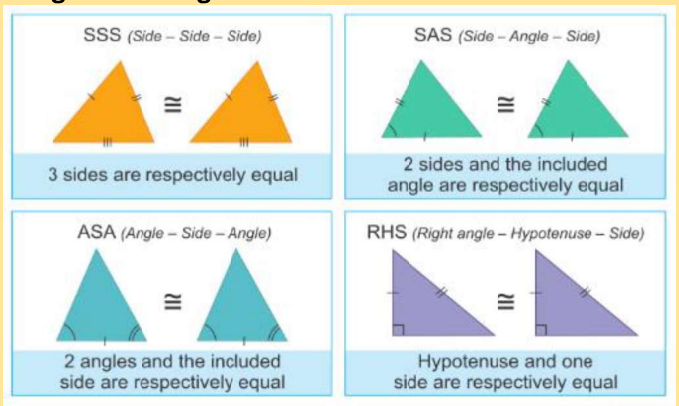
Transformations - enlargement



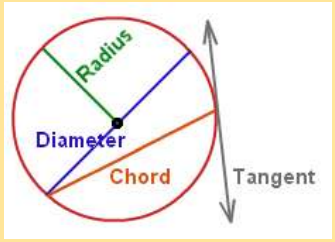
Describe:

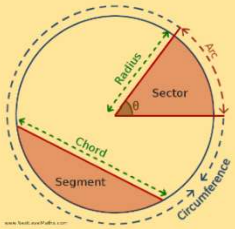
1. It's an enlargement
2. The scale factor (if the image is smaller than the object the scale factor is fractional e.g. ½)
3. The centre of enlargement given as a coordinate

Congruent triangles



Circles





$Area = \pi r^2$

Sector $Area = \frac{\theta}{360}\pi r^2$

$Circumference = \pi d$

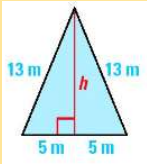
Arc length = $\frac{\theta}{360}\pi d$

Pythagoras' Theorem

$$a^2 + b^2 = c^2$$

Only applies to right angled triangles.

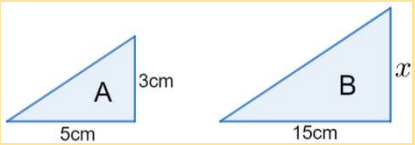
Can be used to find the height of an isosceles triangle



Can be used to find the length distance between two coordinates

Similar shapes

Same shape, different sides
The ratio of the lengths of corresponding sides are equal



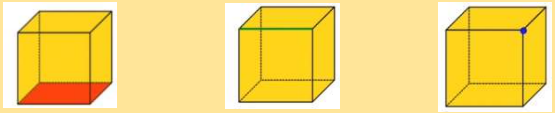
Length scale factor = 15 ÷ 5 = 3

x = 3cm x 3

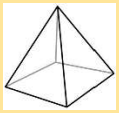
3D notation

<u>Cube:</u>

Faces: 6 Edges: 12 Vertices: 8



<u>Square based pyramid:</u>

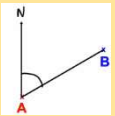


F = 5, E = 8, V = 5

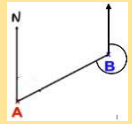
Bearings

Measure from the North
Measured in a clockwise direction
Written using 3 digits

Bearing of B **<u>from A</u>** (start at A)



Bearing of A **<u>from B</u>** (start at B)



Probability and Statistics - Foundation

Averages

Mode: most common piece of data

Mean: Sum of the data ÷ total frequency

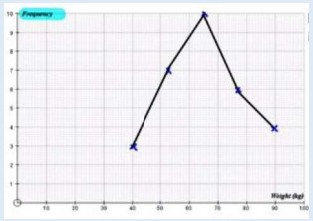
Median: order the data and find the middle value

Range: Highest value – lowest value

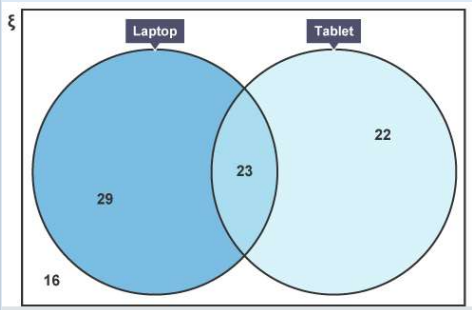
Frequency Polygons

1. Plot frequency at the mid-point
2. Join with straight lines

Weight w (kg)	Frequency
$30 \leq w < 50$	3
$50 \leq w < 55$	7
$55 \leq w < 75$	10
$75 \leq w < 80$	6
$80 \leq w < 100$	4



Venn Diagrams



Information given:
90 pupils were surveyed
52 said they owned a laptop.
45 said they owned a tablet.
23 said they owned both.

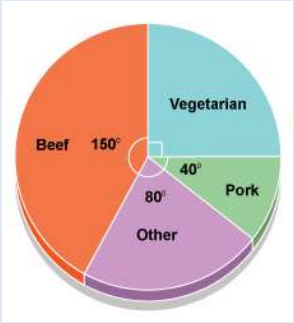
Probability Definitions

Total probability: adds to 1

Relative frequency: $frequency \div total\ trials$

Independent events: one event doesn't impact the other

Reading and Drawing Pie Charts



Find the fraction of the total

1000 people were surveyed

Beef: $\frac{150}{360} \times 1000$

Vegetarian: $\frac{90}{360} \times 1000$

Hair colour	People
Blonde	8
Brown	12
Red	3
Grey	2
Black	6

Find the fraction of the full circle.

Size of Blonde sector: $\frac{8}{31} \times 360°$

Expected outcomes

Expected outcome = probability x number of trials

E.g. A biased spinner is spun 800 times. The probabilities is lands on each colour is below. The probability of it landing on red is the same as the probability of it landing on green. How many times would you expect yellow to come up.

Result	Red	Green	Brown	Yellow
Probability		0.48	0.2	

P(Y) = (1 – 0.48 – 0.2) ÷ 2 = 0.32 ÷ 2 = 0.16

Expected yellow = 0.16 x 800 = 128

Averages from a frequency table

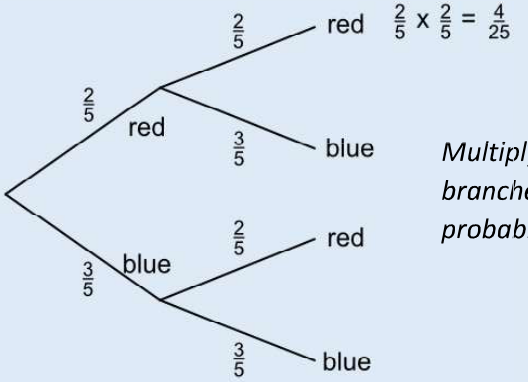
Mean: $\frac{\sum fw}{\sum f}$; where, w is the midpoint of the group.

Median group: find which group the $\frac{n+1}{2}th$, value lies. Where, n is the total frequency.

E.G. in this table 51.5th value which lies in group $8 < w \leq 12$ (using the cumulative frequency

Weight of box (w kg)	Frequency
$0 < w \leq 4$	11
$4 < w \leq 8$	16
$8 < w \leq 12$	29
$12 < w \leq 16$	26
$16 < w \leq 20$	20

Tree diagrams



Multiply along the branches to find each probability.

1. Probability that a red counter is picked both times P(RR) = $\frac{2}{5}$ x $\frac{2}{5}$ = $\frac{4}{25}$
2. Probability that the counters are different colours = P(RB) + P(BR) = $\frac{2}{5}$ x $\frac{3}{5}$ + $\frac{3}{5}$ x $\frac{2}{5}$ = $\frac{12}{25}$